This preprint was generated on *September 17, 2020* at *9:45am* PST

Legal, political science and economics approaches to measuring malapportionment

The U.S. House, the Senate and the Electoral College 1790-2010.

JONATHAN R. CERVAS* Carnegie Mellon University
BERNARD GROFMAN** University of California Irvine

Abstract: ***Objectives***. *We compare and contrast methods for measuring malapportionment from different disciplines: law, political science, and economics.* ***Methods***. *With data from the U.S. House, Senate, and Electoral College over the period 1790-2010, we compare disproportionality measures and compare both across time and between institutions.* ***Results***. *We demonstrate that which approach to measurement we take can dramatically affect some of the conclusions we reach. However, we also demonstrate that the House and the Electoral College are hardly malapportioned, regardless of which measure we use, while the level of malapportionment we observe in the Senate can depend on which measure we use.* ***Conclusions***. *Since there are many axiomatic properties we might wish to satisfy, no one measure is uniformly best with respect to all feasible desiderta. However, one measure, the minimum population needed to win a majority, offers a readily comparable measure across legislatures and jurisdictions, and is easy for non-specialists to understand.*

Data: Surpressed until publication

Many thanks to Esther E. Bailey, Nathan Cisneros, Marek Kaminski, Nicholas Miller, and Aruhn Venkat for useful comments on earlier drafts and general encouragement.

Introduction

A view supported by some distinguished political scientists (see e.g., Lee and Oppenheimer 1999; Dahl 2003) and repeated by journalists (see e.g., Badger 2016), is that the U.S. Senate is inherently undemocratic because of the equal weight given to each state in the Senate despite the vast discrepancy in population across the states. Similarly, it is part of the common wisdom that the Electoral College (EC) is currently highly malapportioned because its two-seat bonus based on Senate seats over-weights small states (see e.g., Toles 2018). In addition to structural malapportionment introduced by the three-fifth's clause of the U.S. Constitutional in the antebellum period, though there are some features of House apportionment that keep it from perfect proportionality, in the post-*Baker v. Carr*, 369 U.S. 186 (1962) era (Ladewig and Jasinski 2008; Ladewig and McKee 2014) the House is nonetheless regarded as providing a level of representation matching population,[1] and the same assessment is generally made for state legislative apportionment.[2] In this essay, we examine these views empirically by comparing malapportionment in the U.S. House, the U.S. Senate and the U.S. Electoral College over the period 1790-2010 by examining multiple metrics coming from law (e.g., the *total population deviation*), political science (e.g., the *Gallagher Index*, Gallagher 1991, and the *Loosemore-Hanby Index*, Loosemore and Hanby 1971), and economics (e.g., the *Gini coefficient*, Lorenz 1905). In order to make our malapportionment measures across the three institutions comparable, we use states as our units. This means that, when we examine the U.S. House of Representatives, we are not interested in questions of within-state variation in district population pre-*Baker v. Carr* or in the manipulation of district populations for partisan purposes (Grofman 1990; Engstrom 2013; McGann et al. 2016).

The most common metrics used by U.S. courts to measure malapportionment across individual districts looks at just two districts, the one most underrepresented and the one most over-represented. We also adapt seats-votes disproportioality measures used in the electoral systems literature to the malapportionment context to look at the full distribution of population values and electoral weights. Our adaptation of the measure of inequality from economics, the *Gini coefficient*, does the same. We show that apportionment equality in the Electoral College looks far more like apportionment equality for the U.S. House of Representatives than it looks like that in the U.S. Senate, regardless of which metric we use. Indeed, when we evaluate EC malapportionment using the two metrics that are most common is the electoral systems literature to measure seats-votes disproportionality, this analysis leads us to the conclusion that the EC behaves with respect to seats to population comparisons much like a proportional representation system does with respect to seats to votes comparisons, though with much

[1] Even the most proportional of allocation methods require rounding into integer values or, as is the case in the U.S. for apportioning the House, may have guaranteed seats for particular types of units regardless of their population. Such rules can create a discrepancy between apportioned seats and actual state population shares. While today's House districts are almost identical in population to one another within any given state, the combination of apportionment rounding rules (the so-called integer allocation problem, Balinski and Young 2001) and the rule that no state can be denied a seat in the House of Representatives regardless of its population, introduces malapportionment into the U.S. House when calculated nationally and not for each state separately.

[2] Malapportionment across states can also occur for the U.S. House of Representatives when Congress fails to fulfill its decennial duty to reapportion the House in accord with new population data. After the 1920 Census, Congress failed to reapportion the House. (see e.g. National Archives – Pieces of History).

smaller deviations. Similarly, a *Gini-index* based measure of EC malapportionment suggests very little bias when we look only at seats share to population comparisons, especially as compared to the vast discrepancies we observe when we look at income distributions.

Moreover, we also show that for both the House and the EC, the time-series data on the magnitude of the malapportionment over the period 1790–2010 is very flat regardless of which measure we use, with some measures of the EC even showing a very minor downtrend in recent elections.[3] In contrast, different metrics lead us to quite different perceptions of changes over time in malapportionment in the Senate. All measures, however, agree that the Senate is far more malapportioned than either the House or the EC (Ladewig and Jasinski 2008).

Background and context

In the U.S., while malapportionment bias is often regarded as inherently undesirable from a normative perspective, it was the perceived effects on government policies stemming from under-representation of city dwellers within states that motivated much of the sentiment that agitated pre-*Baker v. Carr* reformers (see e.g. *Baker v. Carr*, 369 U.S. 186, 1962, *Reynolds v. Sims*, 377 U.S. 533, 1964) (Baker 1955; McCubbins and Schwartz 1988).[4] And, today, while there remains concern for malapportionment in the Senate, the practical foci of current reformers are, on the one hand, on ways to control partisan gerrymandering within states and, on the other hand, on the perceived partisan bias in the EC that is now operating in a pro-Republican direction that leads reformers to seek to replace the EC with a popular vote mechanism for choosing the President or to find other mechanisms that will limit divergence between popular vote and EC outcome.

But malapportionment, in and of itself, may or may not have direct pernicious consequences for the treatment of particular political parties or cognizable groups of voters with distinct interests. For an international example, Singapore has high levels of parliamentary malapportionment, but that malapportionment does not appear to have effects that favor the ruling party, the PAP (Tan and Grofman 2018). In contrast, malapportionment in Japan has historically favored rural areas by over-representing rural voters, and thus been a boon to the the dominant party in Japan, the LDP, whose greatest strength derived from rural voters Moriwaka (2008). [5]

Addressing the partisan consequences of malapportionment is, however, outside

[3] This trend would show less disparity if we were to re-examine the antebellum period removing the provision that apportioned enslaved blacks as only three-fifths of that of all other persons, or adjusted our measures to include a correction based on who was excluded from the franchise.

[4] Relatedly, the failure to reapportion after the 1920 Census was brought about because of reluctance to transfer seats from more rural states whose population was falling, in relative terms, to heavily urban states with growing populations.

[5] Stewart and Weingast (1992) show that in the 19^{th} century "Republicans had manufactured [an] advantage through the strategic admittance of sparsely populated, but strongly Republican, western states. These western "pocket" boroughs provided Republicans with a head start in the Electoral College, and an almost insurmountable lock on the Senate" (as cited in Engstrom (2013, pg. 94)). Moreover, there are other types of effects that malapportionment might produce in addition to direct effects on party representation . Samuels and Snyder (2001, pg. 653), reviewing a number of single country studies, concludes "malapportionment can have an important impact on executive-legislative relations, intra-legislative bargaining and the overall performance of democratic systems."

the scope of this research note. Here we pursue a straightforward and more limited task: assessing the levels of malapportionment in the House, Senate and Electoral College over time and under several different metrics. We regard this as a worthwhile investigation regardless of a linkage (or absence of linkage) between malapportionment and the success of Democratic and Republican candidates for the various offices. We share the view of Dahl (1971) that the "one person, one vote" principle is a necessary component of democratic governance and that, as Taagepera and Shugart (1989) have trenchantly put it, malapportionment is "a pathology". But if we are to assess malapportionment we need to know how to measure it, and we need to recognize when different approaches to measuring malapportionment can yield us very different conclusions about its level.

The structure of the rest of this essay is straightforward. First we introduce the definitions of the seven measures (total population deviation, ratio of largest to smallest district, proportion of population in units with enough seats to command a majority, the Gini Coefficient, 80/20 percentile rank ratio, Loosemore-Hanby index, and Gallagher index). Then we provide graphs showing the empirical values of these seven indices over the period 1790-2010 for the U.S. House, the Senate, and the Electoral College, with some additional information about exactly how values in the various graphs were ascertained. Then we discuss the implications of our findings for both present-day malapportionment and the time-series changes in malapportionment levels in the three national U.S. electorally determined institutions, including a discussion of whether malapportionment in these three institutions has move in synchrony among them.

Measuring Malapportionment

Regardless of whether or why we regard malapportionment as problematic, logically prior is the question: "How do we measure malapportionment?". While there is a 'zoo' of potential measures (Taagepera and Grofman 2003), we focus on a select few which are preeminent in the scholarly and legal literature. It is well recognized that no single measure of disproportionality can capture every feature of interest, and each measure has some desirable properties and some flaws (Cox and Shugart 1991; Monroe 1994; Taagepera, Selb, and Grofman 2014). [6] Similarly we will not seek to discuss which is best other than to note that we believe that different measures pick up different facets of inequality. Thus we disagree with Samuels and Snyder (2001) who reject the use of *total population deviation* as completely inappropriate. While we would not say that courts in multiple countries cannot be wrong, we are not willing to dismiss a court chosen measure of malapportionment out of hand.[7] Instead we seek to compare empirically the results we get from different measures. Our focus will be empirical, looking at the measurement of U.S. political institutions (the House of Representatives, the Senate, and the Electoral College) and what the different measures say about long-term trends and overall magnitudes of disproportionality in each over

[6] The theoretical virtues of the different measures have been extensively investigated (Monroe 1994; Taagepera and Grofman 2003; Karpov 2008; Van Puyenbroeck 2008) and we will not try seek to contribute to the literature on axiomatic comparisons of equality measures.

[7] Edelman (2006) presents an algorithm for minimizing the total population deviation so as to satisfy the court objective function.

the period 1790-2010.[8] Our measures will compare the most common measure used in U.S. Courts, along with other measures proposed by political scientists pre-*Baker v. Carr*, with applications to the population context of those found more recently in the comparative politics literature measuring vote-seat disparities, and of the two common measures of inequality in the economics literature. The degree of concordance among some of our measures is, we believe, rather surprising, as are the results about which measures are most in disagreement with other measures, and how the degree of disagreement among measures varies across the three institutions.

For simplicity of exposition, we present below definitions of all four measures used by courts or proposed by early reformers for use by U.S. courts for the case of single seat constituencies.[9] Let $p_i = persons$ in the i_{th} constituency, $P = total\ population = \sum_{i=1}^{n} p_i$, $\hat{p} = ideal\ population$, i.e., the total population divided by the total number of seats n. Constituencies are indexed by $i = 1, \ldots, n$. p_{max} is the district in the constituency with the largest population, p_{min} is the district in the constituency with the smallest population.[10] Because the central concern that motivated this paper was malapportionment in the Electoral College, for all our calculations we use states as the units. This means that our measure of congressional malapportionment only captures inter-state differences in mean population per House districts analyzed/aggregated at the national level.[11]

Legal Measures of Malapportionment and early Political Science Approaches

The aftermath of *Baker v. Carr* (1962) initially led U.S. federal courts to consider a number of different ways to measure malapportionment (NCSL 2019).[12] The goal was to guarentee "equal representation for equal numbers of people (*Wesberry v. Sanders*, 376 U.S. 1 1964). Rather quickly the U.S. Supreme Court focused on a particular measure, the *total population deviation* (*TPD*, Equation 1)[13], which looks at the relative difference in population between the most underpopulated and the most overpopulated district to the ideal district size. The use of *TPD* is ad hoc, and the court gave no normative justification (Edelman 2015). Among other measures initially proposed by political scientists (see esp. Baker 1966) are the *Voter Equivalency Ratio* (Ladewig 2011)

[8] The issue of malapportioned voting units can be traced at least as far back as the Roman Republic, where voting was by units based on income level, with the wealthy greatly over-represented (Manin 1997).

[9] There are many complexities in defining malapportionment when we move from simple single-seat systems to countries with multi-seat districts and/or a mix of single and multiple seat districts, and/or a tiered system with proportional allocations or compensatory seats in the upper tier (Samuels and Snyder 2001). Because the House and Senate are single seats constituencies, such complications arise only vis-a-vis the Electoral College. There, we weight states by their EC seat-share.

[10] Issues of whether to use measures based on something other than census based population counts, such as total citizen population or eligible voters take us to issues quite distinct from those considered in this paper. See *Evenwel v. Abbott* 578 U.S. __ (2015).

[11] This is equivalent to taking constituency populations within a state to be identical.

[12] It also took a while for there to be definitive legal standards for what levels of malapportionment would be acceptable at different levels of government (NCSL 2019).

[13] Also referred to as *Relative Deviation* or *total maximum deviation*

(*VER*, Equation 2)[14], which is the ratio of the population in the largest district to that in the smallest;[15] the *minimum population share*, which identifies the minimum population needed to control a majority of seats in the legislature; and the *average absolute level of deviation* (Equation 3).

The last of these measures is mathematically equivalent to the *Loosemore-Hanby* (Equation 4) measure of malapportionment; We will reserve discussion of it until the discussion of political science approaches to malapportionment, where we refer to it under the latter title.

The *TPD* measure is conceptually very simple, and like the other three measures it can be used to specify a *de minimis* threshold that can serve as a "bright-line" test for courts.[16] Virtually every other democracy that imposes some form of "one person, one vote" test on its parliamentary constituencies has also adopted a *TPD* based measure, though with widely differing thresholds, most far higher than the ones adopted in the U.S. (e.g., 30% in Germany and 50% in Canada). In the U.S., courts have used different acceptability thresholds for different types of legislatures. For state legislatures, a *TPD* of 10% is usually acceptable, but for Congress the U.S. Supreme Court has pushed for a *TPD* that is effectively zero.[17] See (Handley and Grofman 2008) for a review of legal malapportionment thresholds in many countries.[18]

Total population deviation (TPD) =

$$\frac{(p_{max} \quad p_{min})}{\hat{p}} \tag{1}$$

The TPD is sometimes written as

$$\frac{\hat{p} \quad p_{min}}{\hat{p}} + \frac{\hat{p} \quad p_{max}}{\hat{p}}$$

14 It has also been called the *population deviation ratio*, the *overall range*, *population variance ratio*, *max/min*, and *maximum deviation*.

15 Note that the *total population deviation* measures the absolute difference between seats and votes while the *population deviation ratio* is based on a ratio.

16 Though often called the "total" population deviation (or variance), it might better be called the "maximum" population deviation since it only describes the relationship between the two most extreme units and nothing about the nature of malapportionment in the other constituencies.

17 In *Vieth v. Jubelirer* 541 U.S. 267 (2004), the U.S. Supreme Court rejected a Pennsylvania Congressional redistricting plan on the grounds that it did not adhere to 'one person, one vote' with a deviation of 19 people. The court has been effectively silent on inter-state malapportionment, leaving a potential question open about whether any deviation across states is constitutional. In *U.S. Commerce v. Montana* 503 U.S. 442 (1992), Montana argued that the method of apportionment violated Article I § 2 because under the 1941 law that established the "method of equal proportions". Montana was to lose one of their two seats. Had it retained its two seats, both seats would have been closer to the ideal (national) district population than the one district it had under the apportionment method used. In a 9-0 ruling, the court held that "Congress exercised its apportionment authority within the limits dictated by the Constitution". The Supreme Court summary disposed the district court's ruling in *Clemons v. U.S. Department of Commerce* 710 F. Supp. 2d 570 (N.D. Miss. 2010), which challenged directly inter-state malapportionment, on the grounds of lack of jurisdiction.

18 However, the reader must be careful in interpreting reported thresholds. For example, the threshold in Germany is stated as no more than 15% upwards or downwards from the average, and those who write about Germany may thus correctly characterize it as a 15% tolerance limit but, in our terms, this is a 30% *TPD* value.

The *Voter Equivalency Ratio* is simply the ratio of the largest to the smallest persons per district, with a ratio of 1 indicating no malapportionment.

Voter Equivalency Ratio (VER) =

$$\frac{p_{max}}{p_{min}} \tag{2}$$

Average absolute deviation =

$$\sum \frac{|p_i - \hat{p}|}{n} \tag{3}$$

Finally, to find the *minimum population share* needed to control a majority of the seats in the legislature, for the case of single seat constituencies we order the districts from smallest to largest by population per district. We find the population of the units up to and including the pivotal unit (m) and then divide by the total population to obtain the proportion we seek. To calculate it for the Electoral College, we take the population of each unit in the EC to be equal to each state's population divided by its number of EC seats.[19]

Adapting Political Science Measures of Seats-Votes Discrepancy to the Malapportionment Context

In contrast to the measures used in courts, when students of politics study redistricting, they utilize instead measures of malapportionment adapted from the electoral systems literature on measuring the discrepancy between party vote-share and party seat-share (Samuels and Snyder 2001; Sauger and Grofman 2016, pg. 654).

The *Loosemore-Hanby Index of Distortion* (Equation 4, Loosemore and Hanby 1971) along with the closely related *Gallagher Index* (Equation 5, Gallagher 1991) are the two most common metrics used for measuring seats-votes disproportionality.[20] *Loosemore-Hanby* measures the summed absolute differences between seats and votes, while *Gallagher's Index*, often referred to as a "Least Squares" measure, weights each observation by the size of the deviation, i.e., it squares the deviations. Squaring the deviations puts more weight on larger deviations, while discounting smaller ones. The analogues to these two disproportionality indices in the malapportionment context are shown below.

Loosemore-Hanby Index =

$$\frac{1}{2}\sum_{i=1}^{n} |p_i - \hat{p}| \tag{4}$$

[19] This metric has also been labeled as the "electoral percentage" (Dixon 1968) and the *theoretical control index*, (Grofman and Scarrow 1981). Sometimes the resulting vote proportion is divided by two in order to indicate that only a majority of the votes in each constituency are needed to control the outcome in that constituency, i.e., a party that wins only the barest of majorities in a bare majority of seats in two-party competition can win the election. We will not make use of this normalizing divisor.

[20] There are many other measures that have been proposed.

Gallagher Index =

$$\sqrt{\frac{1}{2} \sum_{i=1}^{n} (p_i \quad \hat{p})^2} \tag{5}$$

Adapting Economic Measures of Equality to the Study of Malapportionment

The economists' use of measures of inequality is most commonly found in the study of income inequality (Yntema 1933; Atkinson 1970; Foster 1985; Bai and Lagunoff 2013). A standard approach in the economic literature on inequality is to report *fractiles* or *percentile ratios*, e.g., the proportion of income held by, say, the richest 80% of the population divided by the proportion of income held by the poorest 20% of the population. Similarly, we can find the the ratio of seat-shares to population shares at the 20^{th} and 80^{th} percentiles.[21] We have chosen to measure the 80^{th} and 20^{th} percentiles for the tables and figures presented in the empirical section of the paper, but the *80/20 ratio* is only intended to be illustrative. It is but one of many ratios we might have used.

The percentile method provides just a crude understanding of malapportionment. The *Lorenz curve*, a graphical tool for displaying inequality first proposed in 1905 by Lorenz (1905) is the natural way to summarize the entire distribution. On a two-dimensional scatterplot, plot the cumulative percentages of the population, on one axis and the cumulative share of income arranged from lowest to highest on the other. This is the *Lorenz curve*. Where all points on the plot are identical, $x = y$, a straight line is drawn, often called the *line of equality*. That is, the top k% of the population holds k% of the income. To provide a single measure derived from a Lorenz curve the *Gini coefficient* is used. It is defined as the ratio of the area of the cumulative frequency distribution and the area below the *line of equality*.[22] Similarly, for a legislature or for the Electoral College, we can plot cumulative population share versus cumulative seat-share.[23]

To create a *Lorenz curve*, order districts such that $v_1 \quad v_2 \quad v_n$ where each district $i = 1, \ldots, n$ gets allocated its share of V, the total vote-share. Individual shares are $v_i = p_i/V$ and $1 = \sum_{i=1}^{n} p_i$. The cumulative proportion of V is then plotted on the x-axis and the cumulative population share on the y-axis. The points start with (0,0) and end at (1,1).

21 This approach dates back to at least 1896, with Vilfredo Pareto's "Cours d'économie politique", where he showed that 80% of the land in Italy was controlled by 20% of the people. The ratio approach in terms of percentile ranks like the ratio approach in terms of largest and smallest district population throws away some of the information about the shape of the distribution *in toto*.

22 This area allows for meaningful comparisons among Lorenz curves which intersect. It can be found through interpolation if we have actual data or can be calculated analytically for different assumed distributional shapes.

23 In the context of economic inequality, the *Gini coefficient* been called the single best measure of inequality (Morgan 1962), but, as noted earlier, we will not attempt to judge measures normatively but rather to assess their degree of concordance when applied to important real world applications. Another approach to equality found in the economic literature is based on voting power using a game theory measure of power such as the *Shapley-Shubik* index or the *Banzhaf index* (Banzhaf 1965; Shapley and Shubik 1954). We will not consider this approach to inequality here.

Empirical Comparisons of Seven Measures of Malapportionment

Natural questions to ask are: "How much and in what ways does the choice of malapportionment measure chosen affect the conclusions we reach about level of malapportionment?", "How have malapportionment levels in the three institutions (the U.S. House of Representatives, the U.S. Senate, and the U.S. Electoral College) we study changed over time?" and, "Are there measures that, while appearing distinct mathematically, tend to give similar answers?".

We will address these questions with U.S. Census data over the period 1790 to 2010. We acknowledge that for any given malapportionment metric, the choice of apportionment method might lead to different calculated levels of malapportionment (Huckabee 2001; Gaines and Jenkins 2009), and that the U.S. has used five different methods of apportionment in its history. The differences do not affect our substantive understanding of malapportionment. To maximize comparability of results over time we apply the "Hill-Huntington", which has been used in the U.S. since the 1940 census (2 U.S.C. 2a), to the entire time-series. For 1790-1990, the source is Gibson and Jung (2002) , for 2000 it is table P003 from the 2000 decennial census, and for 2010 it is from table P3 of the 2010 decennial census. Apportionment for the U.S. House and Electoral College was tabulated in `R`.

We make several simplifying assumptions in order to compare across time.

First, the District of Columbia is not included in either the U.S. House or Senate calculations, and its population is likewise subtracted from the national population figures we use. Only after Amendment XXIII was ratified in the 1960s giving D.C. three electoral votes [24] is D.C. included in the Electoral College measures.[25] After the ratification of this amendment we add D.C. population to the national population total we use for malapportionment calculations for the EC only.

Second, as noted previously, even though different apportionment methods have been used in different census decades, we calculate apportionment using the Hill-Huntington method (*Method of Equal Proportions*), used for apportioning the U.S. House of Representatives and Electoral College since 1941 so as to have consistency over time.[26]

Third, even though the basis of apportionment has changed over time with respect to the inclusion/weighting of African-Americans and Native Americans,[27] we calcu-

[24] D. C. may receive more in the future if its population is sufficiently large to warrant it and no other state has fewer EC seats than it does.

[25] For the purposes of this essay, we do not include the populations of U.S. territories (e.g, Puerto Rico). Though U.S. citizens, these territories currently do not have any voting representation in U.S. political institutions.

[26] After no apportionment in 1920 after a stalemate in Congress, reapportionment was resumed in 1930 and a rule was set in place that provided for automatic reapportionment after in each census in accord with a specified apportionment formula. While that formula was changed for the 1940 census, and a still different formula had been used early in the nation's history, the differences in allocation across apportionment formulae tend to be minor (see Balinski and Young 2001, cf. Janson and Linusson 2012). Though no apportionment was actually done in 1920, we provide the hypothetical 1920 apportionment from the Census population using the Hill-Huntington method.

[27] Article 1, Section 2, Clause 3 of the U.S. Constitution says "Representatives and direct Taxes shall be apportioned among the several States which may be included within this Union, according to their respective Numbers, which shall be determined by adding to the whole Number of free Persons, in-

late apportionment from the summation of the total free population plus three-fifths of the slave population for the U.S. House throughout the entire time period. Of course, after the end of the Civil War, slavery was abolished.

Lastly, because we are most interested in comparisons at the state level in order to facilitate direct comparisons between the House, the Senate, and the Electoral College, despite severe intra-state malapportionment in the U.S. House prior to *Baker v. Carr*,[28], as noted earlier, for the House we treat each district within a state as the state's population divided by the number of members in that state.[29] For all three of the institutions we study, we calculate the ideal population per seat as the total U.S. population divided by the total number of seats; i.e., $idealpopulation = averagepopulation$.[30] Table 1 shows the minimum district, the maximum district, and the ideal district size for each of the three institutions we study.

cluding those bound to Service for a Term of Years, and excluding Indians not taxed, three fifths of all other Persons." Amendment XIV repealed this provision, requiring "representative shall be apportioned . . . counting the whole number of persons in each State. . . ".

[28] See e.g., Altman (1998); Ladewig and Jasinski (2008); Engstrom (2013)

[29] With the exception of Maine and Nebraska, states currently award the state's total Electoral College votes based on the state-wide plurality winner. We use this unit-rule for all states over the entire period.

[30] Two for each state for the U.S. Senate, and for recent decades, 435 for the U.S. House and 538 for the EC. In effect, as noted earlier, we treat the House districts in each state as having an identical population, namely the population of the state divided by the number of House seats allocated to that state.

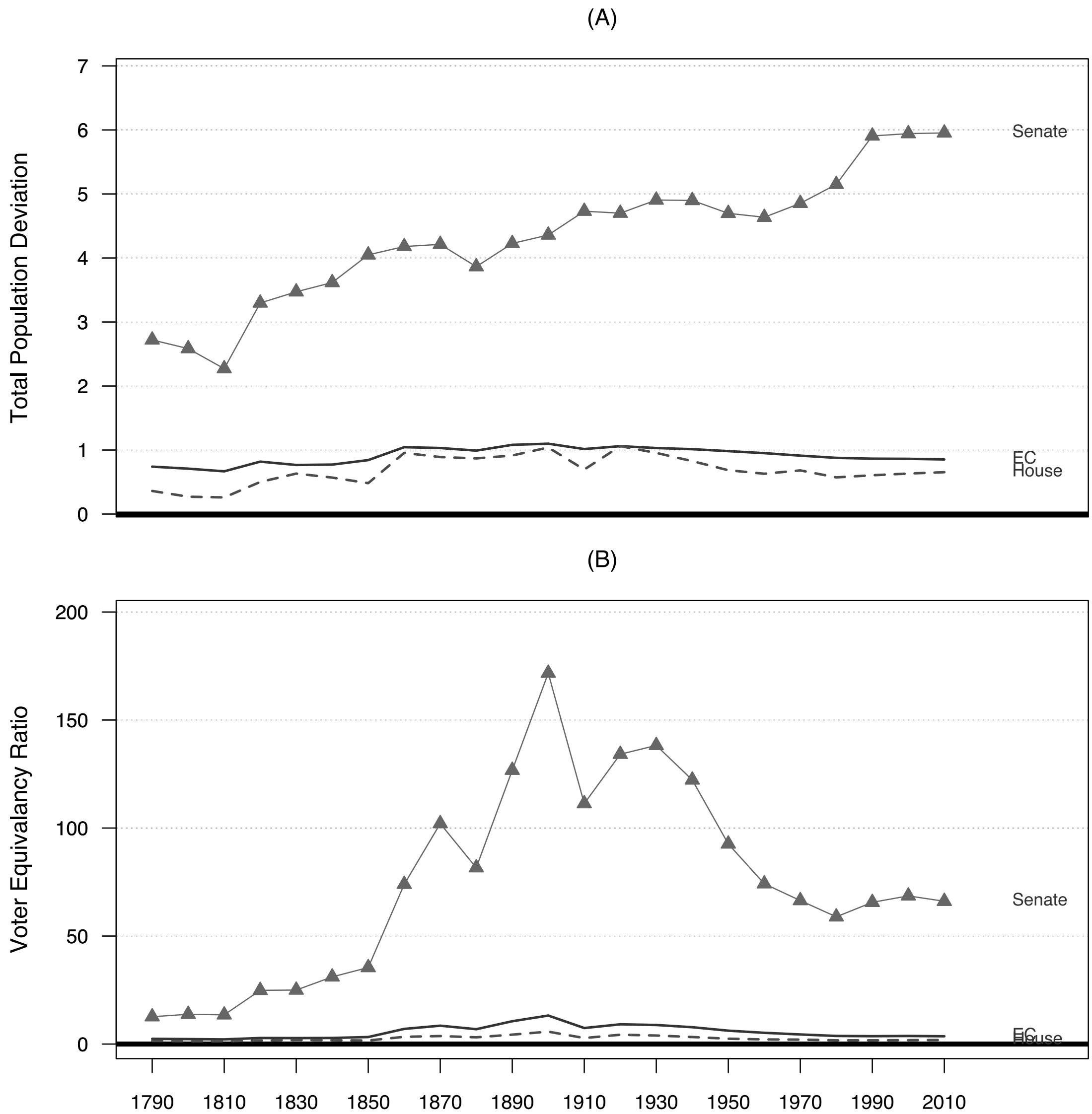
TABLE 1 District Deviation Summaries for the U.S. House, Senate, and Electoral College

Year	U.S. House				U.S. Senate				Electoral College			
	n	*min*	*max*	*ideal*	*n*	*min*	*max*	*ideal*	*n*	*min*	*max*	*ideal*
1790	105	28,475	41,512	36,162	30	29,548	373,805	126,566	135	14,774	35,600	28,126
1800	141	30,893	40,672	36,091	32	32,136	443,074	159,024	173	16,068	36,923	29,415
1810	181	32,966	42,746	37,660	34	36,337	491,576	200,482	215	18,168	39,326	31,704
1820	213	33,293	55,860	45,033	48	27,606	686,406	199,832	261	16,646	46,742	36,751
1830	240	38,374	71,913	53,057	48	38,374	959,304	265,285	288	19,187	53,086	44,214
1840	223	54,415	97,574	75,790	52	39,042	1,214,460	325,021	275	26,028	73,516	61,458
1850	234	73,772	121,305	98,495	62	43,722	1,548,697	371,740	296	29,148	94,777	77,864
1860	241	52,465	175,927	129,245	68	26,232	1,940,368	458,060	309	17,488	122,794	100,803
1870	292	42,941	159,150	130,535	74	21,470	2,191,380	515,082	366	14,314	121,743	104,142
1880	325	62,266	194,327	151,912	76	31,133	2,541,436	649,623	401	20,755	142,763	123,121
1890	356	47,355	206,624	174,156	88	23,678	3,001,587	704,538	444	15,785	166,755	139,638
1900	386	42,335	243,329	193,283	90	21,168	3,634,447	828,969	476	14,112	186,382	156,738
1910	435	81,875	228,398	210,669	96	40,938	4,556,807	954,596	531	27,292	202,525	172,582
1920	435	77,407	334,162	242,007	96	38,704	5,192,614	1,096,594	531	25,802	236,028	198,254
1930	435	91,058	359,611	281,122	96	45,529	6,294,033	1,273,835	531	30,353	267,831	230,298
1940	435	110,247	359,231	301,164	96	55,124	6,739,571	1,364,648	534	36,749	286,790	246,572
1950	435	160,083	395,948	344,587	96	80,042	7,415,096	1,561,408	534	53,361	330,819	282,205
1960	435	226,167	484,632	410,481	100	113,084	8,391,152	1,785,592	538	75,389	392,930	333,314
1970	435	300,382	617,761	465,415	100	150,191	9,976,567	2,024,554	538	100,127	444,804	377,717
1980	435	393,345	690,768	519,328	100	200,926	11,833,951	2,259,075	538	133,950	503,572	421,089
1990	435	453,588	799,065	570,352	100	226,794	14,880,010	2,481,030	538	151,196	551,112	462,286
2000	435	493,782	902,195	645,632	100	246,891	16,935,824	2,808,498	538	164,594	615,848	523,089
2010	435	526,284	989,415	708,377	100	281,813	18,626,978	3,081,438	538	187,875	677,345	573,876

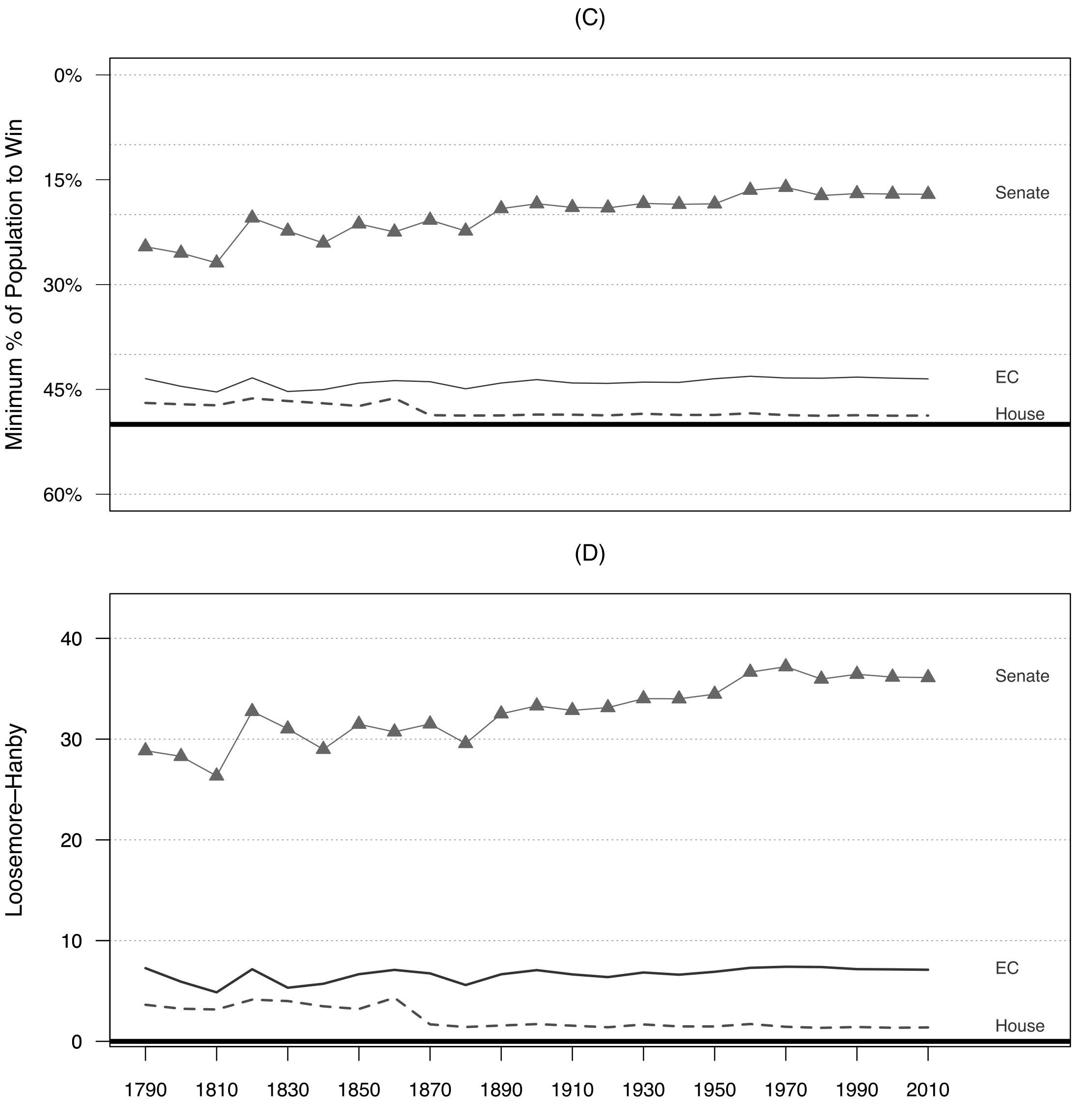
Note: In 1790, there were thirteen states which were apportioned 65 House seats. The district populations [and number of districts] are (ideal = 51,827 [65]; 45,583 [4], 47,484 [3], 47,759 [6], 47,930 [5], 48,035 [7], 51,214 [7], 52,009 [4], 53,528 [11], 54,893 [7], 54,927 [8], 59,440 [1], 71,240 [1], 71,853 [1]).

Figure 1 (A-G) show the comparisons across the three institutions of of single-member districts for each of our seven metrics.

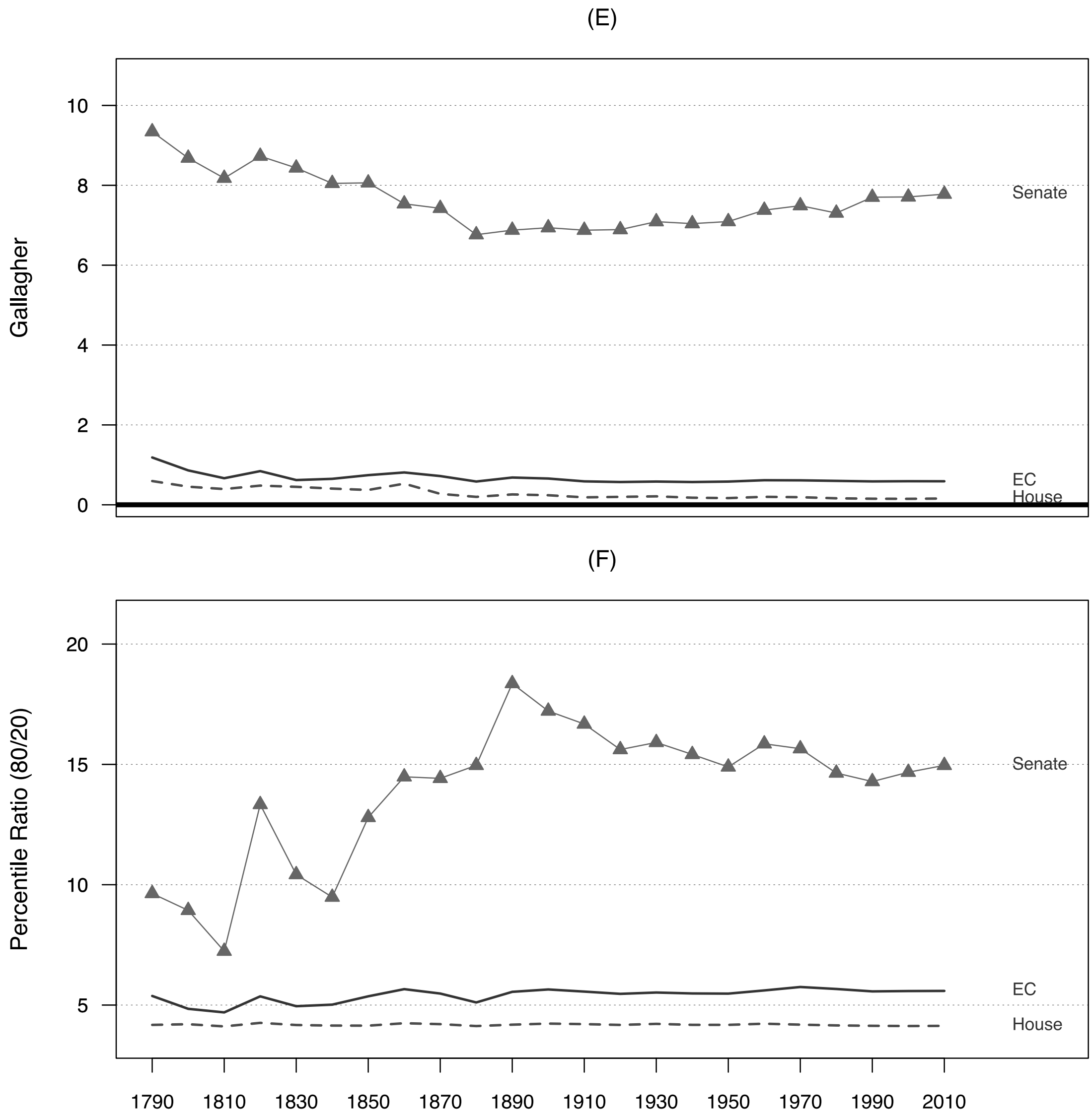
FIGURE 1 Measures of Malapportionment: 1790-2010



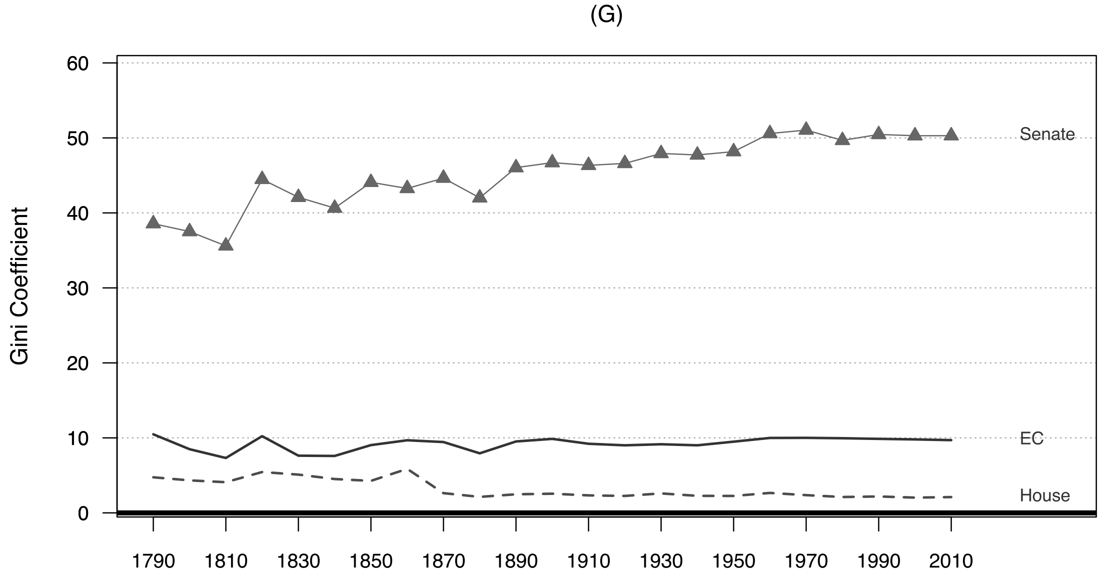
Continued... Measures of Malapportionment: 1790-2010



Continued... Measures of Malapportionment: 1790-2010



Continued... Measures of Malapportionment: 1790-2010



Note: Note: The solid black line indicates perfect voter equality.

All seven of the metrics in Figure 1 support the claim that EC malapportionment is far closer to low levels of House malapportionment than it is to the high levels of U.S. Senate malapportionment.[31] Indeed, if we compare the most recent values we get for those measures to their equivalents in the seats-votes disproportionality context, both the U.S. House and even the U.S. Electoral College exhibit low levels of disproportionality. The numbers shown in Figures 1 D & E, while not as small as the party-based disproportionalities reported for the most highly proportional electoral rules in use world-wide, those of Netherlands[32], and Israel[33] are comparable to the partisan disproportionalities in other western European democracies. For example, tabulating data from Döring and Manow (2017, Table 3: pg. 159) shows that proportional countries have an average *Gallagher* value of 3.89 and majoritarian countries average 12.12. The U.S. House in 2010 was 0.157, the EC was 0.588, and the Senate 7.77.

All seven measures also show a relatively flat pattern of malapportionment for the House and the EC, in general and especially over the past several decades. While, as noted above, the U.S. Senate is far and away the most disproportionate of the three institutions under all measures, unlike what we find for the House and the EC, there are substantial differences across the measures in the over time pattern of Senate malapportionment. For example, the *Total Population Deviation* metric shows the Senate rather steadily exhibiting ever higher levels of malapportionment, though recently leveling off; the *Gini coefficient* and the *Loosemore-Hanby index* show a similar upward pattern, but not as steep, as does the *minimum population share*.[34] But the *Voter Equivalancy Ratio*, in contrast, tends to vary over time, albeit one with the present values still considerably higher than those in the United States' earliest history; the same is true for the *80/20 ratio*, though levels in 2010 are closer to the high levels of the late 19^{th} century than the levels at the founding. Finally, the *Gallagher index* shows first a gradual fall and then a more gradual rise.

We show in Table 2 correlations across our seven measures.

[31] For the *total population deviation*, as a matter of mathematical necessity, as long as the most over-represented and most under-represented state in the EC is the same as their counterpart in the U.S. House, malapportionment in the EC must be larger than malapportionment in the U.S. House.

[32] 2017 Dutch Election: *Loosemore-Hanby* – 1.3, *Gallagher* – 6. Data source: https://www.kiesraad.nl/

[33] 2015 Knesset Election: Loosemore-Hanby – 1.5, Gallagher – 7. Data source: https://www.knesset.gov.il

[34] While voting majorities are not typically determined by state size, this finding rises the prospects of significantly less than 50% of the population controlling the majority of votes in the U.S. Senate.

TABLE 2 Correlations for Seven Measures of Malapportionment

	TPD	VEP	Minimum Winning Population	Loosemore Hanby	Gallagher	Percentile Ratio (80/20)	Gini Index
Voter Equivalancy Ratio	0.90 0.92 0.51	1 1 1					
Minimum Winning Population	0.46 -0.31 -0.90	0.41 -0.13 -0.56	1 1 1				
Loosemore Hanbly	-0.44 0.37 0.88	-0.38 0.18 0.41	-1 -0.98 -0.98	1 1 1			
Gallagher	-0.42 -0.39 -0.57	-0.30 -0.30 -0.86	-0.94 -0.09 0.57	0.95 0.12 -0.43	1 1 1		
Percentile	0.32 0.66 0.71	0.40 0.46 0.83	-0.26 -0.86 -0.82	0.24 0.90 0.71	0.27 -0.19 -0.80	1 1 1	
Gini Index	-0.40 0.33 0.92	-0.34 0.18 0.52	-0.99 -0.97 -0.99	1 0.97 0.98	0.95 0.26 -0.55	0.30 0.84 0.79	1 1 1

Note: The top entry is for the U.S. House, the middle entry is the Electoral College, and the bottom is the U.S. Senate

Since the first two of our measures, *TPD* and *VER*, focus on the same two constituencies (the largest and smallest), we might expect that these two disproportionality measures should correlate highly with one another. They are quite highly correlated for both the U.S. House and the Electoral College, with a correlation exceeding $r = 0.90$ for each institution. But, the same is not true for the U.S. Senate, as the correlation between these two measures is much lower, though still positive at $r = 0.52$. The reduced correlation in the Senate between the two measures is due in part, we believe, to the admission of extremely small states into the union in the mid nineteenth century (Stewart and Weingast 1992; Engstrom 2013), since the ratio measure is even more strongly dependent on extreme values than the difference measure.[35]

Minimum winning population is strongly correlated with the *Gini coefficient* for all three institutions, as is *Loosemore-Hanby*. Similarly, *Loosemore-Hanby* and the *Minimum winning population* are also highly correlated with each other. Similarly, and rather unexpectedly, *TPD* is also highly correlated with all three of these other measures. Thus, whatever the differences in the axiomatic properties of these three metrics, in practice, at least for the time-series data on the three U.S. institutions we examine, they tend to move in parallel. In factor analytic terms, these three measures scale on the same dimension. We find a similar pattern of correlation for the *80/20 ratio* but, despite the high linear correlations, when we examine Figure 1 we see that, for the Senate, it yields a non-monotonic pattern and thus we should be cautious of the results of linear correlation analysis.

In contrast, *Gallagher* and *VER* exhibit divergent patterns with the other measures for some of our three institutions. We might have expected *Loosemore-Hanby* and *Gallagher* to be highly correlated since they are are very similar in mathematical form, but empirically their correlation varies by institution. For the U.S. House, they are highly correlated (0.92); for the EC, they are likewise highly correlated (0.78); for the U.S. Senate, they are more negatively correlated (0.41). More generally, The *Gallagher* measure is negatively correlated with all the other measures (with the exception of *minimum winning population*, for the Senate, which has a positive correlation of $r = 0.54$) for both the Senate and the Electoral College, but positively correlated for the House. We believe that this difference in strength of correlation across institutions reflects the discounting by *Gallagher* of the contribution to malapportionment of very small jurisdictions, a factor which plays a more important role in shaping values in the Senate and the EC than it does in the House.

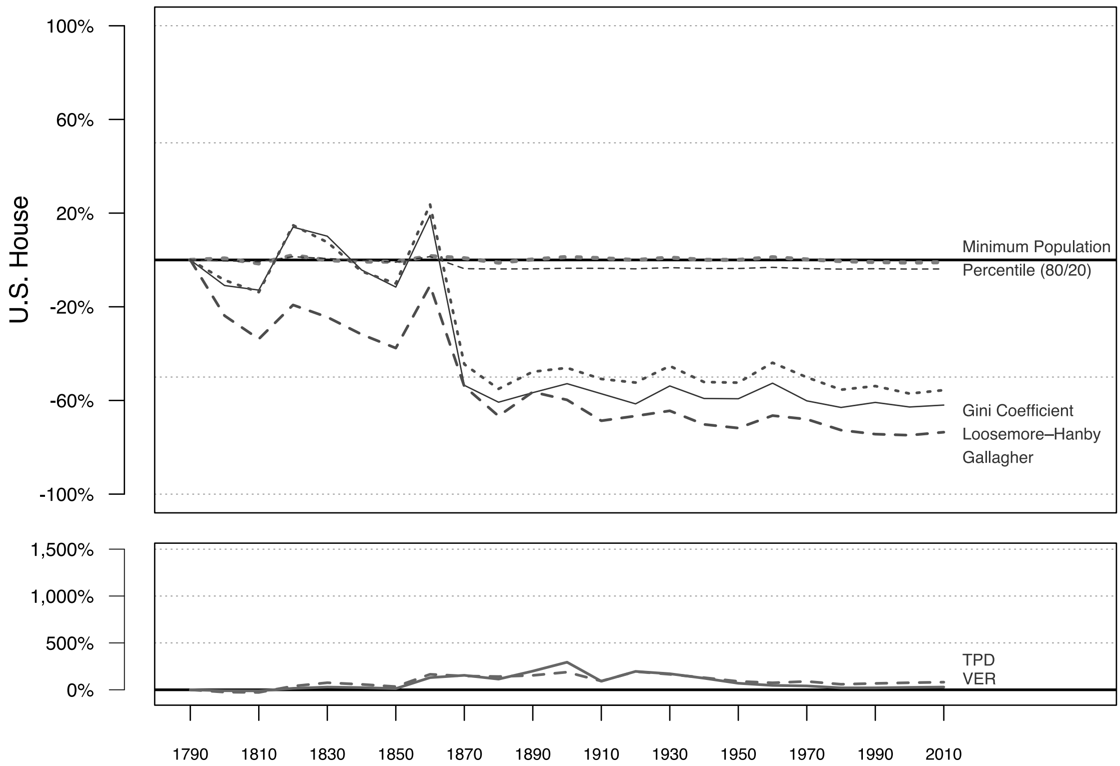
Discussion

The goals of this essay have (1) been to address the magnitude of bias derived from the purely mechanical effects of rules determining the relationship between seat-share and state population share using time-series data for three important U.S. institutions, and (2) to assess the degree to which measures of inequality/disproportionality common in the disciplines of law, political science, and economics, when adapted to the malapportionment context, yield different answers to determining malapportionment inequality over time for the three institutions.

Our principal finding is a clear one: in practice, Electoral College malapportion-

[35] For instance, Nevada entered the union in 1864 with a census total population of 6,857. However, Nevada does not enter our calculations until 1870 with population of 42,291.

FIGURE 2 Percent Change in Malapportionment from U.S. Founding, 1790-2010 – U.S. House



Note: Because of the wide disparities for the *Max/Min* and *TPD* measure compared to the other five, we have created separate plots on different scales.

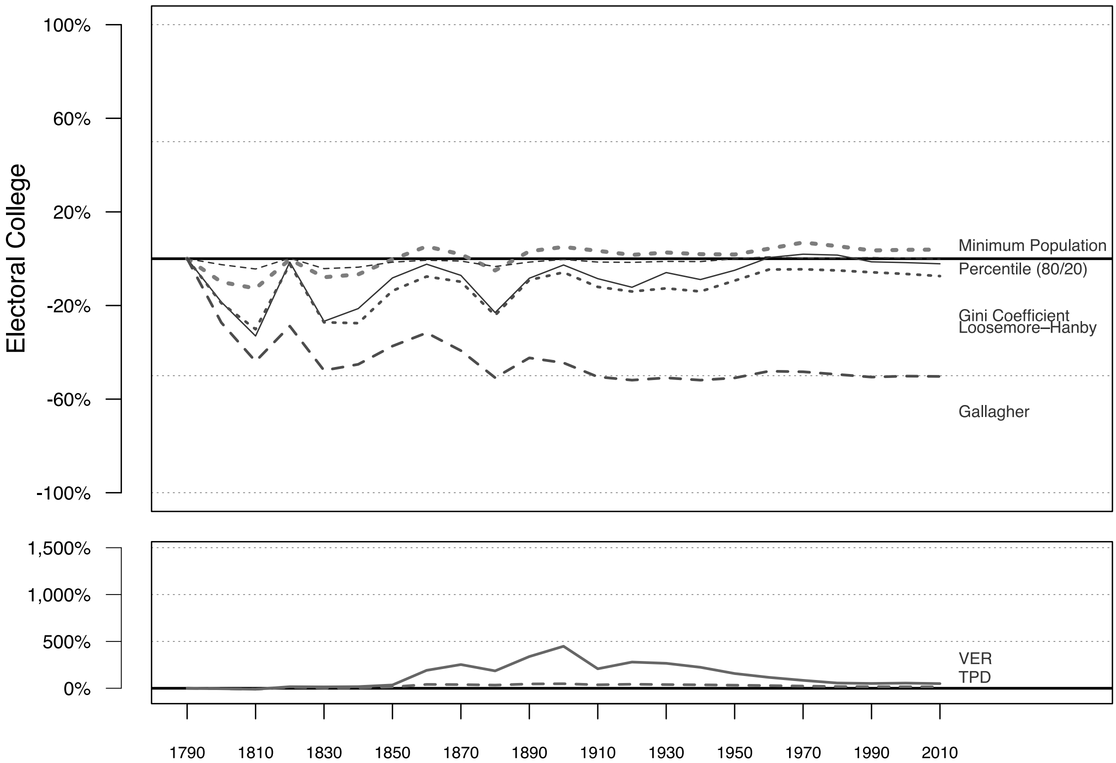
ment is not very much larger than malapportionment in the U.S. House of Representatives and EC malapportionment is far lower than the malapportionment we find in the U.S. Senate.[36] Indeed, as we noted earlier in the text, the numbers for EC for the *Loosemore-Hanby* and *Gallagher* indices shown in Figure 1 d & e, while not as small as the party-based disproportionalities between votes and seats reported for the most highly proportional electoral rules in use world-wide, are comparable to the partisan votes-seats disproportionalities in other western European democracies.

Moreover, the analyses we present above enable us to demonstrate that, despite the freezing of the House size and the logical presumption that malapportionment in the EC should therefore increase, the discrepancy between popular vote outcome and EC outcome that occurred in 2000 and 2016 cannot be blamed on an increasing EC malapportionment in recent decades.[37] Contemporary levels of EC malapportionment are, by virtually all measures, presently at or slightly below previous levels.

[36] The Electoral College may be regarded as essentially a mixture between an upper and a lower chamber, but far more closely resembling the latter. Samuels and Snyder (2001) offer analysis of malapportionment in a comparative perspective which shows that malapportionment levels in upper chambers are characteristically much greater than in lower chambers.

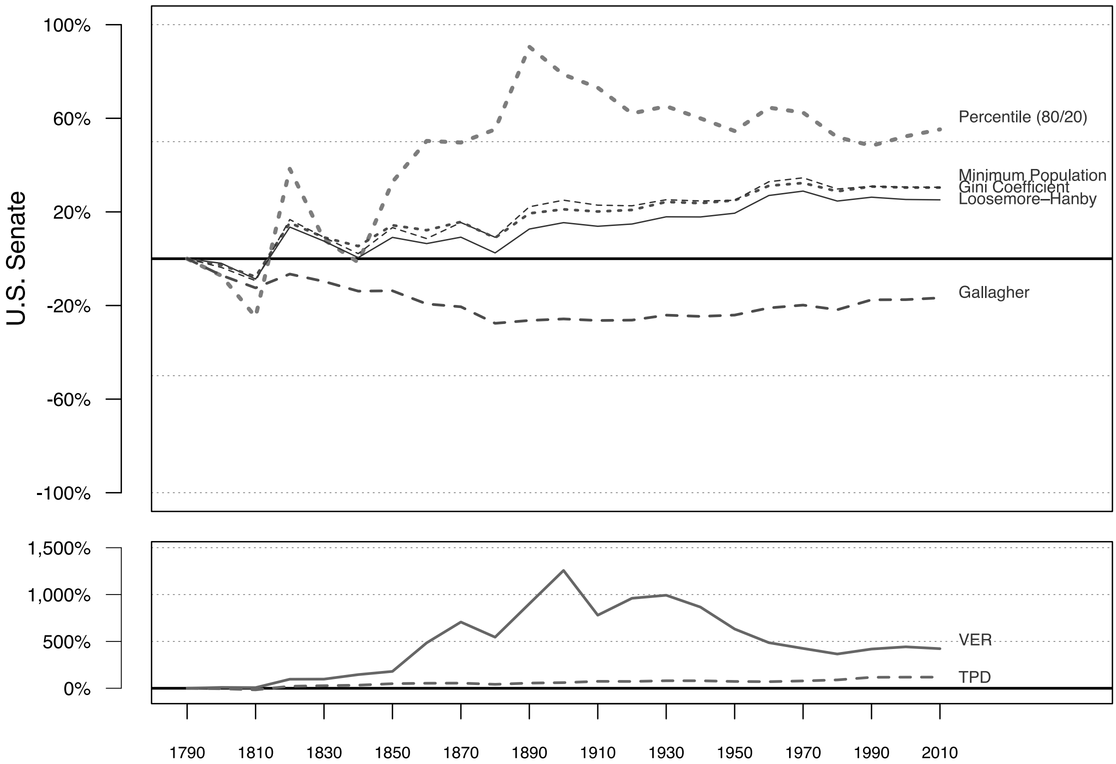
[37] It can be shown that increasing the U.S. House by a reasonably small number would have led to a Gore victory (Neubauer and Zeitlin 2003; Barthélémy, Martin, and Piggins 2014).

FIGURE 2 Continued... Electoral College



Note: Because of the wide disparities for the *Max/Min* and *TPD* measure compared to the other five, we have created separate plots on different scales.

Figure 2 Continued... U.S. Senate



Note: Because of the wide disparities for the *Max/Min* and *TPD* measure compared to the other five, we have plotted them separately on different scales.

Our second major set of findings has to do with the degree of agreement among our various malapportionment measures. We have examined this concordance in terms of graphs showing whether malapportionment has increase, decreased, or stayed flat since the country's founding under our various measures; and we have also looked at linear correlations among our seven measures. Figure 2 presents the same data as in Figure 1, but in a way that faciliates comparisons across our different metrics. From Figure 2 we see that: (a) Comparing 2010 and 1790, for the U.S. House, all of the measures show closely comparable values in the two years (with the partial exceptions of *TPD* and *80/20 ratio* which show slight increases). (b) The same is true for the seven measures of malapportionment in the Electoral College. (c) In contrast, for the U.S. Senate, six of our seven measures show higher levels of malapportionment in 2010 than in 1790, while *Gallagher* shows a downwardly sloping pattern (Figure 1 e), Furthermore, (d) unlike the monotonicity, whether positive or negative, found for the Senate in the other five measures, *VER* and the closely related *80/20 ratio* exhibits non-monotonic patterns for the Senate over the period 1790-2010. (e) Five of our seven measures, all but *Gallagher* and *VER* correlate very strongly with one another and can, in factor analytic terms, be considered as scaling on a single dimension for all three institutions, though we place less reliance on the linear correlation for the *Percentile (80/20)* measure because of the non-monotonic pattern it exhibits for the U.S. Senate. (f) However, with respect to comparability across measures, we regard our results about the *Gallagher index* as equally important. While, in the context of seats-votes disproportionalities, *Gallagher* is a metric that has recently been given a great deal of favorable attention, its lack of strong correlations with other measures, with negative correlations for two of our three institutions, suggests a sharp note of caution vis-a-vis its use in the context of malapportionment.[38]

Conclusions

One point we wish to highlight is the fact that, throughout this article, we have eschewed evaluating the measures whose applications we review. There are good reasons for this reluctance. First, some measures, such as *total population deviation*, even though lacking in clear justification, have a long track record of use by courts in the U.S. and elsewhere. Second, since there are many axiomatic properties we might wish any measure to satisfy, it is clear from work on the axiomatic foundations of disproportionality measures that there is no measure which is uniformly best with respect to all feasible desiderata. Third, the choice of measures can be expected to differ with intended use. Nonetheless, to the extent that we have a preference, it is for measures that reflect the full range of district-specific values rather than only extreme values. Moreover, among such measures the one that we personally find most appealing is the *minimum population* needed for control of the legislature. This measure has the great advantages of being readily comparable across legislatures and being easy for non-specialists to understand.

Another point we would wish to emphasize is the difference we find between malapportionment in the House and Senate. If the most malapportioned state in the

[38] Gallagher (1991) provides comparisons of the *Gallagher index* (which he refers to as the *least squares index*) with other measures in the context of national level seats-votes relationships. In that context he finds considerable similarity in results across measures.

House is a state with a single representative, then that state is also the most malapportioned state in the Senate with respect to overrepresentation. But, while the largest state can be many many times the population of the smallest state, the population of largest state is much larger than the mean size of the House district in that state, since that value is the population of the state divided by the number of House seats allocated to that state. Thus, using some metrics, malapportionment in the Senate is expected to be worse than malapportionment in the House, without regard to the overall amount of equality in the institution.

However, because of the peculiarities of apportionment rules, and the associated rounding process, the most overrepresented states in the EC need not be the state with the smallest population, and the same is true for the most underrepresented state. Nonetheless, as we have seen from Figure 1 not only is the Senate much more malapportioned than the House, but also the degree of malapportionment in the Senate has been growing for all measures except the *Gallagher index*, while the same is not true for the House. Thus, if there is a malapportionment problem that is stark, it is the malapportionment in the Senate. Figure 2 shows that after the Civil War the House became much more proportional. In the U.S. Senate, ca. 2010, 50% of the votes are controllable by 17.1% of the population (see Figure 1C) In contrast, in the House, a controlling majority is potentially held by 48.7% of the people [39] If we turn to the Electoral College, whose supposedly out of whack malapportionment level was the original motivation for writing this articles, a controlling majority is potentially held by 43.5% of the people, so the differences in malapportionment between the House and the EC is not that big according to this measure. And this is true for virtually all of our malapportionment measures.

Note that our results fly in the face of the common wisdom about how badly malapportioned the EC supposedly is. The explanation for the mismatch between expectations and reality is two-fold. First malapportionment in the EC is roughly a weighted average of malapportionment in the House and malapportionment in the Senate, with the House having by far the greater weight. Second, in the EC, the states with greatest seats to population discrepancy (the smaller states) do not make up a large share of the EC vote. The U.S. Senate presents a far more serious challenge to our understanding of the majoritarian principle of democracy than does the Electoral College. When a noted democratic theorist Dahl (2003) asks, "How Democratic Is the American Constitution?," at least with respect to malapportionment, it is the Senate rather than the Electoral College for which this question is most relevant.

But that is not to say that continued use of the Electoral College does not pose issues of political fairness. The basic reason why EC malapportionment effects are commonly overstated is the confusion between population based malapportionment and seats-votes disproportionality. To understand Electoral College effects, we need to distinguish the mechanical effects of the Electoral College that we may think of as "malapportionment related" from effects that are tied to the *geographic* distribution

[39] Senate malapportionment is often taken to be matter out of anyone's control, while the House is usually taken to be fine with respect to population equality after the *one person, one vote* revolution. But, the latter conclusion leaves out the continuing differences in mean district size across states (for example, Ladewig (2011), shows that interstate malapportionment can exceed 9,000% of the level of intra-state malapportionment found unconstitutional). However, the analyses we present here are based on inter-state malapportionment rather than intra-state malapportionment so this problem does not arise.

of the votes across states in each election. The first type of bias due to discrepancies between a state's EC vote-share and the state's population or House delegation share which arise simply because EC vote allocations equal the size of a state's U.S. House delegation plus the size of the state's U.S. Senate delegation. The former applies throughout any given redistricting decade; the latter is election specific. In addition to the partisan distribution of voters across states, turnout differences among the states may also operate to bias outcome so as to create a discrepancy between the popular vote winner and the EC winner. These election specific effects can be substantial enough to generate a partisan bias that can lead to a divergence between popular vote majority winner and the winner of the Electoral College vote (Cervas and Grofman 2019). Calculations of this bias suggests that it has sometimes favored Democrats and sometimes favored Republicans, but with a large pro-Republican bias in 2016. (Grofman, Koetzle, and Brunell 1997; Pattie and Johnston 2014; Zingher 2016) [40] Likewise, partisan gerrymander introduces in the House partisan bias, where the minority party can control the majority of votes with sometimes substantially fewer votes.

[40] A third factor that could matter is the size of the House. In 2000, as Neubauer and Zeitlin (2003) point out, a larger House size might have given the election to Gore but, given the magnitude of Trump's EC victory, the House size would have had to have been increased by an implausible amount in order to switch the EC outcome in 2016 (Cervas and Grofman 2019). Considering the relative importance of different reasons for EC and popular vote discrepancies is beyond the scope of this study.

References

Altman, Micah. 1998. "Traditional Districting Principles: Judicial Myths vs. Reality." *Social Science History* 22(2): 159.

Atkinson, Anthony B. 1970. "On the measurement of inequality." *Journal of Economic Theory* 2(3): 244–263.

Badger, Emily. 2016. "As American as Apple Pie? The Rural Vote's Disproportionate Slice of Power - The New York Times." New York Times New York Times.

Bai, Jinhui H., and Roger Lagunoff. 2013. "Revealed Political Power." *International Economic Review* 54(4): 1085–1115.

Baker, Gordon E. 1955. *Rural versus urban political power: The nature and consequences of unbalanced representation*. Random House.

Baker, Gordon E. 1966. *The reapportionment revolution: Representation, political power, and the Supreme Court*. Random House.

Balinski, Michel L., and H. Peyton Young. 2001. *Fair Representation: Meeting the Ideal of One Many One Vote*. Second ed. Washington D.C.: Brookings Institution Press.

Banzhaf, John F. III. 1965. "Weighted Voting Doesn't Work: A Mathematical Analysis." *Rutgers Law Review* 19(2): 317–343.

Barthélémy, Fabrice, Mathieu Martin, and Ashley Piggins. 2014. "The architecture of the Electoral College, the House size effect, and the referendum paradox." *Electoral Studies* 34: 111–118.

Cervas, Jonathan R., and Bernard Grofman. 2019. "Are Presidential Inversions Inevitable? Comparing Eight Counterfactual Rules for Electing the U.S. President*." *Social Science Quarterly* 100(4): 1322–1342.

Cox, Gary W., and Matthew Soberg Shugart. 1991. "Comment on Gallagher's 'proportionality, disproportionality and electoral systems'." *Electoral Studies* 10(4): 348–352.

Dahl, Robert A. 1971. *Polyarchy : participation and opposition*. Yale University Press.

Dahl, Robert A. 2003. *How democratic is the American Constitution?* Yale University Press.

Dixon, Robert G. 1968. *Democratic Representation: Reapportionment in Law and Politics*. New York: Oxford University Press.

Döring, Holger, and Philip Manow. 2017. "Is Proportional Representation More Favourable to the Left? Electoral Rules and Their Impact on Elections, Parliaments and the Formation of Cabinets." *British Journal of Political Science* 47(1): 149–164.

Edelman, Paul H. 2006. "Minimum Total Deviation Apportionments." In *Mathematics and Democracy*. Berlin, Heidelberg: Springer Berlin Heidelberg , 55–64.

Edelman, Paul H. 2015. "Voting power apportionments." *Social Choice and Welfare* 44(4): 911–925.

Engstrom, Erik J. 2013. "The Partisan Impact of Malapportionment." In *Partisan Gerrymandering and the Construction of American Democracy*. University of Michigan Press , 148–164.

Foster, James E. 1985. "Inequality measurement." *Fair allocation* 33: 31–68.

Gaines, Brian J., and Jeffery A. Jenkins. 2009. "Apportionment matters: Fair representation in the US house and electoral college." *Perspectives on Politics* 7(4): 849–857.

Gallagher, Michael. 1991. "Proportionality, disproportionality and electoral systems." *Electoral Studies* 10(1): 33–51.

Gibson, Campbell, and Kay Jung. 2002. "Historical Census Statistics On Population Totals by Race, 1790 To 1990, And By Hispanic Origin, 1970 To 1990, for The United States, Regions, Divisions, And States." Unpublished paper, note. `https://www.census.gov/content/dam/Census/library/working-papers/2002/demo/POP-twps0056.pdf`

Grofman, Bernard. 1990. *Political gerrymandering and the courts*. Eds. ed. Agathon Press.

Grofman, Bernard, and Howard A. Scarrow. 1981. "The riddle of apportionment: Equality of what?" *National Civic Review* 70(5): 242–254.

Grofman, Bernard, William Koetzle, and Thomas L. Brunell. 1997. "An Integrated Perspective on the Three Potential Sources of Partisan Bias: Malapportionment, Turnout Differences, and the Geographic Distribution of Party Vote Shares." *Electoral Studies* 16(4): 457–470.

Handley, Lisa, and Bernard Grofman. 2008. *Redistricting in Comparative Perspective*. Oxford University Press.

Huckabee, David C. 2001. The House of Representatives Apportionment Formula: An Analysis of Proposals for Change and Their Impact on States.

Technical report Congressional Research Service. https://www.everycrsreport.com/files/20010810{_}RL31074{_}141690aef23aca45fb289624e019d8e2a1130d2.pdf

Janson, Svante, and Svante Linusson. 2012. "The Probability of the Alabama Paradox." *Applied Probability Trust* 49(3): 773–794.

Karpov, Alexander. 2008. "Measurement of disproportionality in proportional representation systems." *Mathematical and Computer Modelling* 48(9-10): 1421–1438.

Ladewig, Jeffrey W. 2011. "One Person, One Vote, 435 Seats: Interstate Malapportionment and Constitutional Requirements Is Our Constitutional Order Broken - Structural and Doctrinal Questions in Constitutional Law: Gerrymandering, Congressional Representation, and Trust in the Poli." *Connecticut Law Review School of Law* 43(4).

Ladewig, Jeffrey W., and Mathew P. Jasinski. 2008. "Remedies for Interstate Malapportionment of the U.S. House of Representatives." *Perspectives on Politics* 6(1): 89–107.

Ladewig, Jeffrey W., and Seth C. McKee. 2014. "The Devil's in the Details: Evaluating the One Person, One Vote Principle in American Politics." *Politics and Governance* 2(1): 4.

Lee, Frances E., and Bruce Ian. Oppenheimer. 1999. *Sizing up the Senate : the unequal consequences of equal representation*. University of Chicago Press.

Loosemore, John, and Victor J. Hanby. 1971. "The Theoretical Limits of Maximum Distortion: Some Analytical Expressions for Electoral Systems." *British Journal Political Science* 1(1): 467–477.

Lorenz, M. O. 1905. "Methods of measuring the concentration of wealth." *Publications of the American Statistical Association* 9(70): 209–219.

Manin, Bernard. 1997. *The Principles of Representative Government*. Cambridge: Cambridge University Press.

McCubbins, Mathew D., and Thomas Schwartz. 1988. "Congress, the Courts, and Public Policy: Consequences of the One Man, One Vote Rule." *American Journal of Political Science* 32(2): 388.

McGann, Anthony J., Charles Anthony Smith, Michael Latner, and Alex J. Keena. 2016. *Gerrymandering in America: The House of Representatives, the Supreme Court, and the Future of Popular Sovereignty*. Cambridge University Press.

Monroe, Burt L. 1994. "Disproportionality and malapportionment: Measuring electoral inequity." *Electoral Studies* 13(2): 132–149.

Morgan, James. 1962. "The Anatomy of Income Distribution." *The Review of Economics and Statistics* 44(3): 270.

Moriwaka, Toshimasa. 2008. "The Politics of Redistricting in Japan." In *Redistricting in Comparative Perspective*, eds. Bernard Grofman, and Lisa Handley. eds ed. Oxford University Press , 107–114.

NCSL. 2019. *Redistricting Law 2020*. Legislatures, National Conference of State.

Neubauer, Michael G., and Joel Zeitlin. 2003. "Outcomes of Presidential Elections and the House Size." *PS: Political Science & Politics* 36(4): 721–725.

Pareto, Vilfredo. 1896. *Cours d'économie politique*. Tome premi ed. Lausanne: .

Pattie, Charles, and Ron Johnston. 2014. ""The electors shall meet in their respective states": Bias and the US Presidential Electoral College, 1960–2012." *Political Geography* 40: 35–45.

Samuels, David, and Richard Snyder. 2001. "The Value of a Vote: Malapportionment in Comparative Perspective." *British Journal of Political Science* 31(4): 651–671.

Sauger, Nicolas, and Bernard Grofman. 2016. "Partisan bias and redistricting in France." *Electoral Studies* 44: 388–396.

Shapley, Lloyd S., and Martin Shubik. 1954. "A Method for Evaluating the Distribution of Power in a Committee System." *American Political Science Review* 48(3): 787–792.

Stewart, Charles, and Barry R. Weingast. 1992. "Stacking the Senate, Changing the Nation: Republican Rotten Boroughs, Statehood Politics, and American Political Development." *Studies in American Political Development* 6(2): 223–271.

Taagepera, Rein, and Bernard Grofman. 2003. "Mapping the Indices of Seats–Votes Disproportionality and Inter-Election Volatility." *Party Politics* 9(6): 659–677.

Taagepera, Rein, and Matthew Soberg Shugart. 1989. *Seats and votes: the effects and determinants of electoral systems*. Yale University Press.

Taagepera, Rein, Peter Selb, and Bernard Grofman. 2014. "How Turnout Depends on the Number of Parties: A Logical Model." *Journal of Elections, Public Opinion & Parties* 24(4): 393–413.

Tan, Netina, and Bernard Grofman. 2018. "Electoral rules and manufacturing legislative supermajority: evidence from Singapore." *Commonwealth & Comparative Politics* 56(3): 273–297.

Toles, Tom. 2018. "Democrats are now forbidden by law from controlling more than one half of one branch of government - The Washington Post." Washington Post Washington Post.

Van Puyenbroeck, Tom. 2008. "Proportional representation, gini coefficients, and the principle of transfers." *Journal of Theoretical Politics* 20(4): 498–526.

Yntema, Dwight B. 1933. "Measures of the Inequality in the Personal Distribution of Wealth or Income." *Journal of the American Statistical Association* 28(184): 423.

Zingher, Joshua N. 2016. "The relationship between bias and swing ratio in the Electoral College and the outcome of presidential elections." *Journal of Elections, Public Opinion and Parties* 26(2): 232–252.